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THE SCRAMJET

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Flight of an object through the atmosphere at high speed is equivalent to having air flow past the object at the same speed. The ramjet utilizes the relative velocity of the engine through the atmosphere to produce thrust. The kinetic energy of the flowing air is converted to a substantial pressure rise through the diffuser. Heat is then added (fuel plus combustion) and the hot high pressure flow is re-expanded to free stream static pressure. The thrust of the ramjet is then

$$T = \dot{m}\left[U_{ex}\left(1 + \frac{f}{a}\right) - U_o\right]$$

$$= \rho U_o^2 A\left[\frac{U_{ex}}{U_o}\left(1 + \frac{f}{a}\right) - 1\right] \tag{1}$$

$\dot{m}$ is the mass flow rate through the diffuser, U_{ex} is the jet speed at free stream static pressure, U_o is the flight speed, f/a is the fuel-air ratio, ρ the free stream density and A the stream tube cross sectional area.

The one dimensional flow relations for pressure, temperature, and Mach number are

$$\frac{P}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}} \tag{2}$$

and

$$\frac{T}{t} = \left(1 + \frac{\gamma - 1}{2} M^2\right) \tag{3}$$

Where P and T are stagnation pressure and temperature and p and t are static values; M is the Mach number and γ is the ratio of specific heats.

For an ideal ramjet with no losses and no change in fluid properties, both the total and static pressures of the free stream will equal those of the expanded exhaust flow. Hence from equation (2) the exhaust Mach numbers will equal the free stream Mach numbers and from equation (3)

$$\tau = \frac{T_{ex}}{T_o} = \frac{t_{ex}}{t_o} \tag{4}$$

but

$$\frac{U_{ex}}{U_o} = \frac{M_{ex}}{M_o} \cdot \sqrt{\frac{t_{ex}}{t_o}} = \tau \tag{5}$$

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Hence

$$T = \rho U_o^2 A\left[\left(1 + \frac{f}{a}\right)\sqrt{\tau} - 1\right] \tag{6}$$

If there are losses through the engine due either to poor diffusion and friction or due to the heat addition processes, the total pressure of the exhaust will be less than that of the free stream even though the static pressures are the same. For equal static pressures at the exhaust and in the free stream, application of equation (2) yields

$$\left(\frac{M_{ex}}{M_o}\right)^2 = \frac{\left(\frac{P_{ex}}{P_o}\right)^{\frac{\gamma-1}{\gamma}}\left(1 + \frac{\gamma - 1}{2} M_o^2\right) - 1}{\frac{\gamma - 1}{2} M_o^2} \tag{7}$$

Also

$$\left(\frac{U_{ex}}{U_o}\right)^2 = \left(\frac{M_{ex}}{M_o}\right)^2 \cdot \frac{t_{ex}}{t_o} = \left(\frac{M_{ex}}{M_o}\right)^2 \frac{T_{ex}}{T_o} \cdot \left(\frac{P_o}{P_{ex}}\right)^{\frac{\gamma-1}{\gamma}} \tag{8}$$

by successive use of equations (2) and (3). Thus, from (7) and (8):

$$\left(\frac{U_{ex}}{U_o}\right)^2 = \frac{2\tau}{(\gamma - 1)M_o^2}\left[1 + \frac{\gamma-1}{2} M_o^2 - \left(\frac{P_o}{P_{ex}}\right)^{\frac{\gamma-1}{\gamma}}\right] \tag{9}$$

Hence from equation (1) the ramjet thrust is

$$T = \rho U_o^2 A\left\{\left(1 + \frac{f}{a}\right)\sqrt{\frac{2\tau}{(\gamma - 1)M_o^2}\left[1 + \frac{\gamma - 1}{2} M_o^2 - \left(\frac{P_o}{P_{ex}}\right)^{\frac{\gamma-1}{\gamma}}\right]} - 1\right\} \tag{10}$$

Equation 10 of course, reduces to equation (6) under conditions of no loss in total pressure.

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For the subsonic combustion ramjet flying at a given set of ambient conditions, the static temperature and pressure of the flow entering the combustion chamber increase rapidly with Mach number. The following table includes the stagnation temperature and pressure ratios for discrete Mach numbers as well as the static temperatures and pressures for combustion chamber entrance Mach numbers of 0.2 and 1.0.

	Stagnation Conditions		Combustion Mach Number = 0.2			Combustion Mach Number = 1.0		
Mo	P/p_o	T/t_o	p/P_o	t/t_o	A/A_o	p/p_o	t/t_o	A/A_o
1	1.895	1.2	1.84	1.19	0.338	1.0	1.0	1.0
2	7.82	1.8	7.60	1.78	.571	4.13	1.5	1.69
3	36.7	2.8	35.7	2.78	1.434	19.4	2.33	4.24
4	152	4.2	147.8	4.16	3.62	80.3	3.5	10.7
5	529	6.0	514	5.95	8.3	279	5.0	25
6	1580	8.2	1535	8.13	17.9	835	6.83	53
9	21 150	17.2	20 580	17.05	115	11 170	14.3	327
12	144 400	29.8	140 000	28.6	432	76 400	24.8	1276

For combustion chamber entrance Mach number 0.2, the static temperatures and pressures are nearly equal to stagnation conditions. The values are reduced by the Mach numbers one ratio for combustion chambers entrance Mach numbers of 1.0 conditions. The rapid increase in static pressure implies a structural requirement to contain these pressures. The rapid increase in temperature generates extreme cooling problems as well as difficulties in providing materials of sufficient strength and corrosion resistance.

Also of interest is the observation of the acute diffusion problem if the ramjet is to fly over a range of Mach numbers. If isentropic diffusion were possible, the free stream flow area would have to be contracted to the sonic or minimum area ratio, then expanded subsonically to the desired combustion chamber entrance Mach number. Isentropic diffusion for a range of flight Mach numbers from 1 to 12 would thus require the absurdly difficult range of diffuser area ratios of 1 to 1276. The geometry variations required at high Mach numbers for the subsonic combustion ramjet therefore become excessive.

Furthermore, the rise in static temperature at the entrance to the combustion chamber may be so great at hypersonic speeds for the subsonic combustion ramjet that combustion in the usual sense does not occur. Injection of the fuel might even cool off the stream, and many of the combustion radicals remain dissociated. The reaction products recombine only after the flow has been sufficiently expanded in the exhaust nozzle to lower the static temperature of the stream to levels that permit the recombination reactions to take place. Thus, part of the supersonic flow in the diffuser, the subsonic portion of the diffuser, the combustion chamber, and the low supersonic portion of the exhaust nozzle could perhaps be eliminated from the engine

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without impairment to the combustion efficiency. Such a design modification would have the further effect of eliminating many of the losses associated with the severe diffusion requirements. However, combustion at supersonic speed is required.

There will be no problem of maintaining combustion in a supersonic stream under proper conditions. Hydrogen, when properly mixed with air, is spontaneously combustible at temperatures above 1850° R. The hypersonic ramjet can easily maintain such static temperatures in the flow even though the flow entering the combustion chamber is supersonic. Reaction distances are less than a foot, yielding reasonable combustion chamber lengths. Furthermore, at hypersonic flight speeds, the static pressure rise is so great that the total pressure loss associated with supersonic combustion which would outlaw supersonic combustion at lower flight Mach numbers may no longer be crucial. The employment of supersonic combustion actually improves the overall engine performance at flight speeds above about Mach 6.

With supersonic combustion, the entrance Mach numbers to the combustion chamber will increase with flight speed. The maximum entrance Mach number is established by the requirement that the static temperature be above 1850° R. On the other hand, the static temperature should be kept as low as the combustion process will permit to prevent dissociation and to permit as large a temperature rise as possible due to combustion upstream of the exhaust nozzle. Hence, the 1850° R nearly sets the minimum entrance Mach number to the combustion chamber as well.

The stagnation temperature in the free stream and at the entrance to the combustion chamber are equal. Hence from equation (3), the combustion chamber static temperature is

$$\frac{t_c}{t_o} = \frac{1 + \frac{\gamma - 1}{2} M_o^2}{1 + \frac{\gamma - 1}{2} M_c^2} \tag{11}$$

or

$$M_c^2 = M_o^2 \frac{t_o}{t_c} + \frac{2\left(\frac{t_o}{t_c} - 1\right)}{\gamma - 1} \tag{12}$$

If we make the assumption that $t_c = 1850^\circ$ R and $t_o = 460^\circ$ R then $t_c/t_o = 4$ and the relation for the combustion chamber Mach number become

$$M_c^2 = \frac{1}{4}\left(M_o^2 - 15\right) \tag{13}$$

Thus at $M_o = 6$, $M_c = 2.3$; at $M_o = 12$, $M_c = 5.7$. Clearly the requirement to diffuse only down to Mach 5.7 at $M_o = 12$ is much less stringent than to diffuse to Mach 1.

The mass flow at the combustion chamber entrance must be the same as in the free stream. Hence:

$$\rho_o U_o A_o = \rho_o U_o A_o \tag{14}$$

or

$$\frac{p_o M_o A_o}{\sqrt{t_o}} = \frac{p_c M_c A_c}{\sqrt{t_c}}$$

but

$$\frac{p_c}{p_o} = \frac{P_c}{P_o}\left\{\frac{1 + \frac{\gamma - 1}{2} M_o^2}{1 + \frac{\gamma - 1}{2} M_c^2}\right\}^{\frac{\gamma}{\gamma-1}} = \frac{P_c}{P_o}\left(\frac{t_c}{t_o}\right)^{\frac{\gamma}{\gamma-1}} \tag{15}$$

Hence the free stream to combustion chamber area ratio is

$$\frac{A_o}{A_c} = \frac{M_c}{M_o} \cdot \frac{P_c}{P_o} \cdot \left(\frac{t_c}{t_o}\right)^{\frac{\gamma}{\gamma-1} - \frac{1}{2}} = \frac{M_c}{M_o} \cdot \frac{P_c}{P_o} \cdot \left(\frac{t_c}{t_o}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \tag{16}$$

As we have assumed, t_c/t_o is nearly constant and of value 4.

Hence:

$$\frac{A_o}{A_c} \simeq 64 \frac{M_c}{M_o} \cdot \frac{P_c}{P_o} \tag{17}$$

A relation is now required for the pressure recovery ratio. A diffuser parameter K_D is frequently used in Scramjet work which presumably is nearly independent of flight Mach number. The quantity K_D is related to kinetic energy efficiency and total pressure recovery ratio by the relations

$$\eta_k = K_D + \left(1 - K_D\right)\left(\frac{M_c}{M_o}\right)^2 \frac{t_c}{t_o} = 1 + \frac{2}{(\gamma - 1)M_o^2}\left[1 - \left(\frac{P_o}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right] \tag{18}$$

Inserting M_c^2 from equation (12) into equation (18) yields the following relation:

$$\left(\frac{P_o}{P_c}\right)^{\frac{\gamma-1}{\gamma}} = \frac{t_c}{t_o} - \left(\frac{t_c}{t_o} - 1\right) K_D \tag{19}$$

For the special case where $t_c/t_o = 4$, equation (19) becomes

$$\left(\frac{P_o}{P_c}\right)^{\frac{\gamma-1}{\gamma}} = 4 - 3K_D \tag{20}$$

Thus for the Scramjet, the assumption of constant static temperature at the entrance to the combustion chamber and a constant value of K_D in the diffuser leads to the conclusion that the pressure recovery ratio in the diffuser is independent of Mach number and is constant. Hence the variation in required diffuser geometry arises by equation (17) only through the Mach number ratio. From equation (13), the Mach number ratio is a slowly varying function, changing from 0.382 for $M_o = 6$ to 0.475 at Mach 12. The ratio is 0.5 for $M_o = \infty$. These numbers suggest that diffuser geometry variations of only about 25 percent would be required for the speed range from Mach number 6 to 12. In contrast, the subsonic combustion ramjet would require a variation of 2400 percent. When the fact is recognized that constant combustion chamber entrance static temperature is only the approximate mode of operation, constant geometry engine may be comtemplated for the speed range from Mach 6 up to Mach 12 and above.

This mode of operation also results in a combustion chamber static pressure that is independent of flight Mach number; both factors in equation (15) are of course constant. Hence the stringent pressure loads that would be expected for the subsonic combustion ramjet do not occur. In general, flight attitudes are chosen to give static pressures in the combustion chamber on the order of one atmosphere. Hence the job of cooling the combustion chamber structure will be simpler in spite of the severe heat transfer rate.

Also of interest is the calculation from equation (12) of the entrance velocity to the combustion chamber.

$$\frac{U_c}{U_o} = \frac{M_c}{M_o} \cdot \sqrt{\frac{t_c}{t_o}} = \sqrt{1 + \frac{2\left(1 - \frac{t_c}{t_o}\right)}{(\gamma - 1)M_o^2}} \tag{21}$$

or for $t_c/t_o = 4$,

$$\frac{U_c}{U_o} = \sqrt{1 - \frac{15}{M_o^2}} \tag{22}$$

Clearly the velocity ratio increased toward unity as the flight speed increases, but the velocity change decreases. At Mach 6, the combustion chamber velocity is about 1422 feet per second below that of the free stream. At Mach 12 this velocity difference is only 626 feet per second. Thus at Mach 12, the flow would be decelerated from 12,000 feet per second down to 11,374 feet per second at the entrance to the combustion chamber. Burning would take place and a stagnation pressure loss would occur due to combustion at high speeds. The resulting flow would be accelerated in the exhaust nozzle to the ultimate exhaust jet velocity. The performance estimates may be continued through the combustion chamber and exhaust nozzle by means of one dimensional equations. The approach is conventional and will not be continued here. The derivations that were included are to illustrate the essential differences and advantages of the supersonic combustion ramjet over the subsonic combustion device for hypersonic flight.

In summary, employment of supersonic combustion offers the promise of (a) very reasonable pressures in the combustion chamber (of order one atmosphere) hence the structural and cooling problems are simplified; (b) relief from the problems associated with high temperature dissociation of the fluid and combustion problems; (c) relief from the severe variable geometry requirements of the inlet and exhaust nozzle that are so characteristic of the subsonic combustion ramjet, and (d) elimination of the special requirements (such as normal shock waves) associated with having both supersonic and subsonic flow fields; (e) elimination of many loss mechanisms through the simplified diffuser requirement, and (f) simplification of engine design and improvement of engine efficiency.

Of course, at hypersonic speeds there will be severe problems associated with aerodynamic heating. Cowl leading edges must be cooled. Hence they will have bluntness. For smaller size engines (5 ft. D) the pressure drag due to the bow wave ahead of the blunt edges may be too great. Hence cooled leading edges will probably include sweep back to decrease the pressure drag. The rise of external contractions and expansions in the inlet and exhaust nozzle design respectively are desired to minimize surfaces that must be cooled.